Labor Income Share Dynamics with Variable Elasticity of Substitution

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Anatomy of the Accumulation Principle

Labor income share ↓ with

Labor income share ↑ with

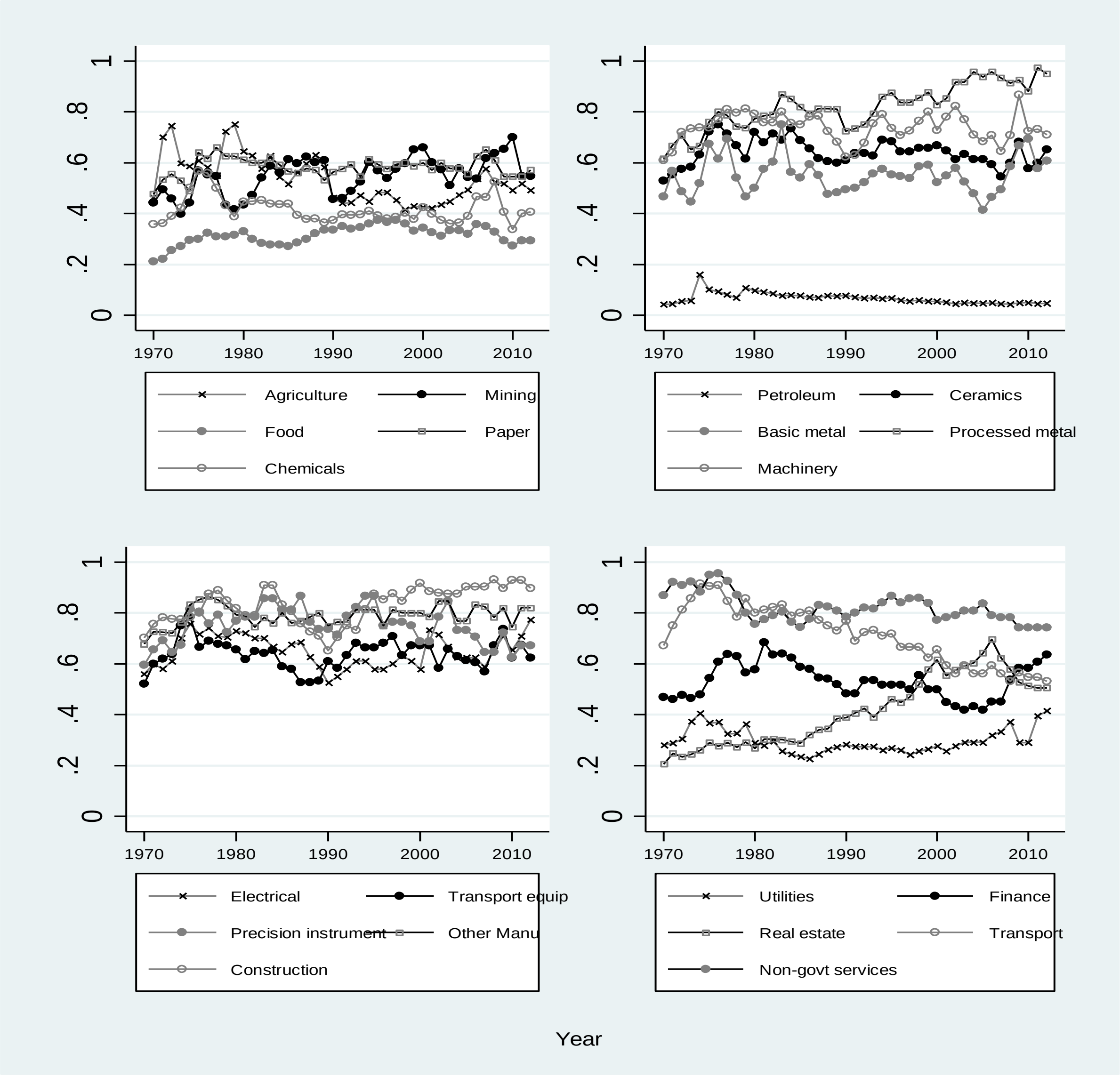
A decline in the relative price of capital goods (Karabarbounis and Neiman, 2014) if $\sigma_{KL} > 1$

Or

An increase in the capital-output ratio (Piketty, 2014). if $\sigma_{KL} < 1$

σ_{KL} = elasticity of substitution between capital and labor.

Labor income share trends in Japanese industrial sectors

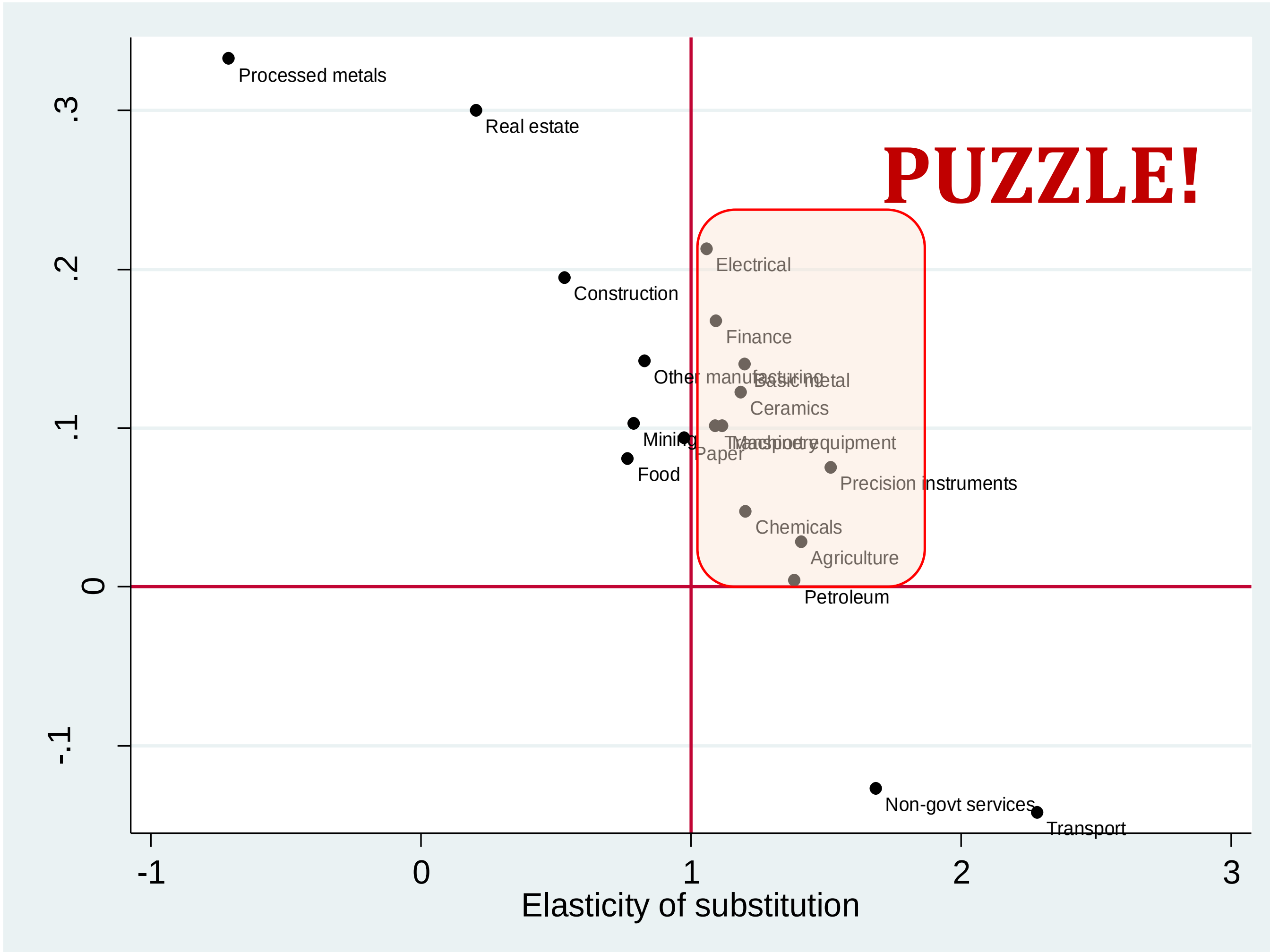


Labor income share increased (by more than 10 pp) in 12 out of 20 sectors between 1970-2012.

Capital-output ratio increased in most of these sectors between 1970-2012.

Source: Author's calculation based on the Regional-level Japanese Industrial Productivity (R-JIP) database

Labor income share ⬆ with $\sigma_{KL} > 1$ **in a CES model**



Source: Author's calculation based on the Regional-level Japanese Industrial Productivity (R-JIP) database

The goal of this paper

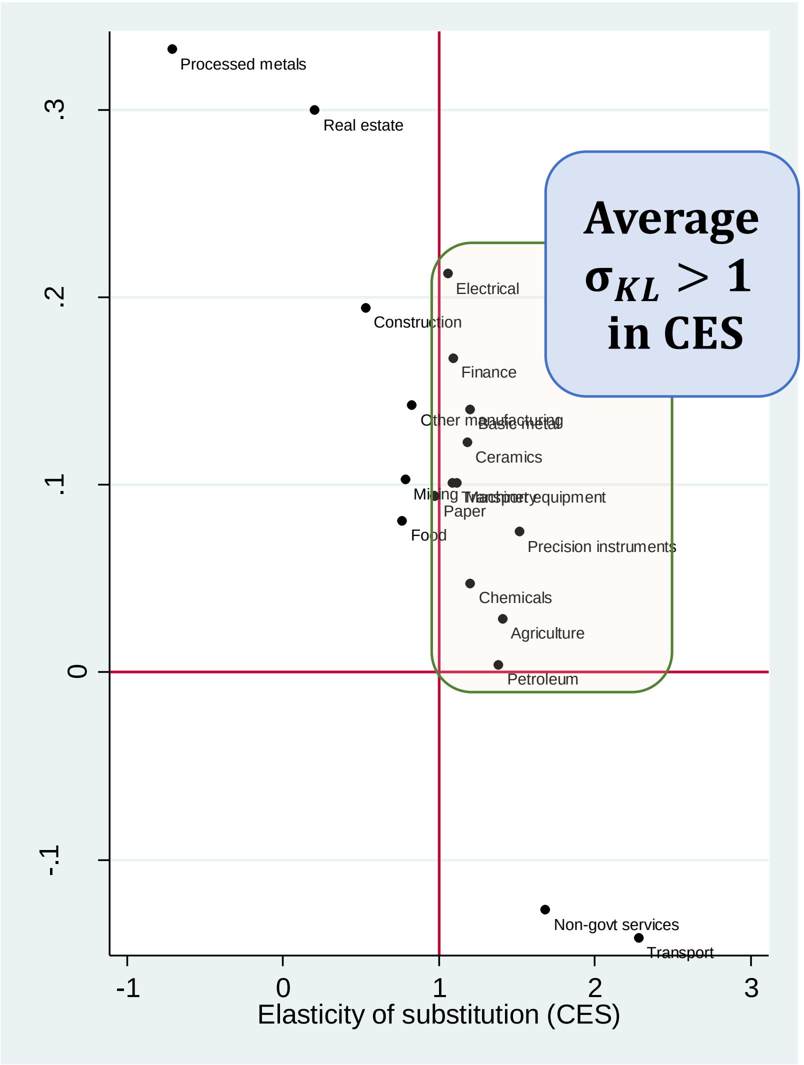
1. Develops a variable elasticity of substitution (VES) framework and evaluates its performance in explaining labor income share dynamics relative to the CES model.

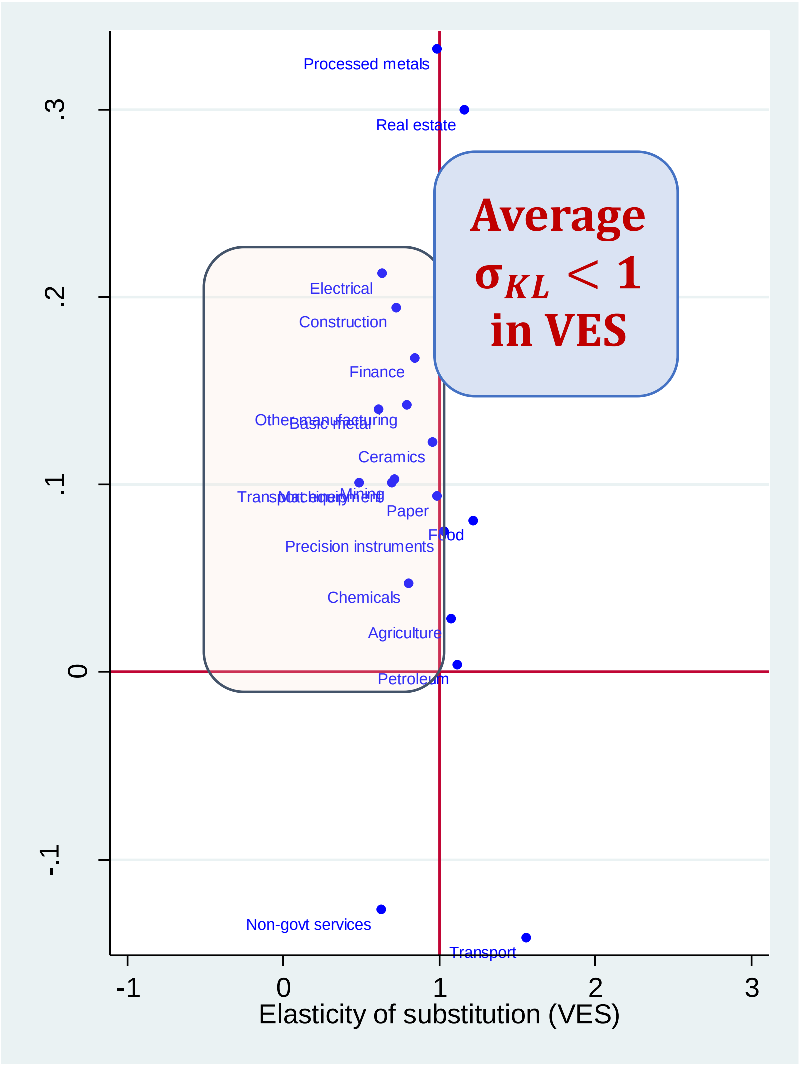
Result → Both CES and VES model calibrate the actual data well

2. Examines if a VES framework explains the accumulation principle more closely than the CES model

Result → Average $\sigma_{VES} < 1 < \sigma_{CES}$ (predominantly)

The main contribution





The rest of the slides

1. Data

2. Why VES? Empirical validity to a variable σ_{KL}.

3. A VES production framework.

4. Estimation of structural parameters in the CES and VES model.

5. Compare the descriptive accuracy and predicting power of the CES and the VES models.

1. Data and labor income share trends across 20 Japanese sectors

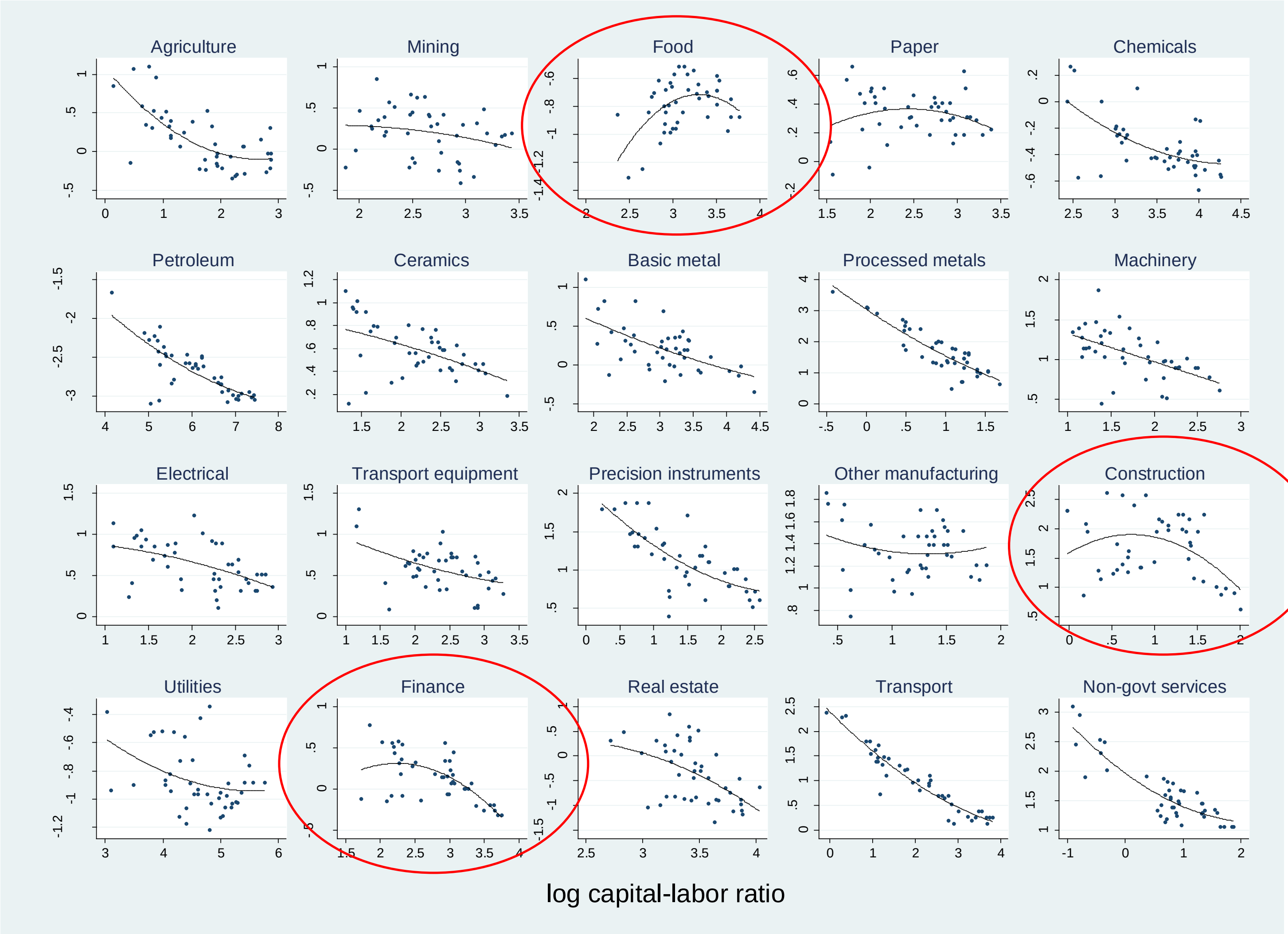
Regional Japan Industrial Productivity (R-JIP) data

The R-JIP database compiles value-added output in current and constant prices, quality-adjusted labor input and capital input for all 23 industrial sectors for every year from 1970 to 2012.

Following Fukao and Perugini (2018), I construct the labor income share by sector (industry) as the ratio of nominal total labor compensation to nominal value added (at current prices).

For measurement issues, I restrict this analysis to 20 main industrials sectors after dropping textiles, wholesale and retail trade and private services.

2. Why a VES model? Motivation



Source: Author's calculation based on the Regional-level Japanese Industrial Productivity (R-JIP) database

2. Why a VES model? A formal test

$$(1)\ log\frac{Y}{L} = \alpha + \varepsilon logW + \beta log\frac{K}{L} + u$$

(Liu and Hildebrand, 1965)

$\boldsymbol{\beta = 0}$ → the CES production function (Arrow et al., 1961)

$\boldsymbol{\beta \neq 0}$ → VES production functions (Revankar, 1967;

Industry code	Industry name	$\hat{\beta}$ (coefficient of $\log\frac{K}{L}$)	R2	Breusch-Godfrey Statistic	F-test ($\hat{\beta}$ = 0)
1	Agriculture	0.210***	0.949	13.395	29.92***
2	Mining	0.209***	0.951	21.933	32.28***
3	Food	0.166***	0.970	23.294	11.35***
5	Paper	0.123***	0.989	5.315	18.9***
6	Chemicals	0.277***	0.982	19.097	59.13***
7	Petroleum	0.487***	0.972	6.472	169.54***
8	Ceramics	0.177***	0.995	6.703	119.39***
9	Basic metal	0.226***	0.982	13.285	100.21***
10	Processed metals	0.127***	0.994	5.471	89.76***
11	Machinery	0.151***	0.989	16.594	56.21***
12	Electrical	0.210***	0.985	25.351	53.72***
13	Transport equipment	0.182***	0.984	21.696	39.53***
14	Precision instruments	0.152***	0.995	6.084	216.07***
15	Other manufacturing	0.128***	0.996	10.195	44.58
16	Construction	0.157***	0.991	21.29	80.67***
17	Utilities	0.086	0.937	25.782	1.89
19	Finance	0.343***	0.991	7.642	188.45***
20	Real estate	0.201**	0.655	32.306	5.32**
21	Transport	0.165***	0.990	6.123	262.53***
22	Non-govt services	0.124***	0.997	20.945	105.21***

Source: Author's calculation based on the Regional-level Japanese Industrial Productivity (R-JIP) database

3. A VES production framework

Why not the "**normalized CES technology**" (La Grandville, 1989; Klump and La Grandville, 2000; Klump et al, 2007)?

- in the normalization process, the share parameter arises when the capital-labor ratio equals to a benchmark level

- an arbitrary choice of the benchmark capital-labor ratio does not reduce the parameter space to one dimension

I work with an extended version of the Lu and Fletcher (1968) model, with the share parameter defined as a cumulative distribution function of a 3-parameter Weibull distribution of the capital-labor ratio.

3. A VES production framework contd..

The **VES-W** model → $V_t = Ae^{\mu t}\left[\theta_t K_t^{\frac{\varepsilon-1}{\varepsilon}} + (1-\theta_t)\left(\frac{K}{L}\right)_t^{\frac{-\beta}{\varepsilon}} L_t^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$

The parameters ε and β are estimated using Equation 1 (Lu, 1967).

A time-variant distribution parameter, θ_t, measures the relative weights of factors in the production of output, V.

$$\theta_t \simeq F(k_t|\alpha,\gamma,\pi) = e^{-\left(\frac{\mathrm{k}_t-\gamma}{\pi}\right)^{\alpha}}$$

θ_t are drawn from the cumulative density functions of the Weibull distribution of $k(=\frac{K}{L})$, with scale parameter α, shift parameter γ, and shape parameter π.

3. A VES production framework contd..

The Weibull (3 parameter) model shows the best fit.

Comparison of the Kolmogorov-Smirnov statistics for six models (Frechet with 2 and 3 parameters, General extreme value, Gumbel max and Weibull 2 and 3 parameters) from the class of generalized extreme value distribution with that of the normal distribution.

			Frechet (2 pars)	Frechet (3 Pars)	General Extreme Value	Gumbel Max	Normal	Weibull (2 pars)	Weibull (3 pars)
1	Agriculture	Statistic	0.147	0.110	0.123	0.129	0.159	0.114	0.089
		Ranking	6	2	4	5	7	3	1
2	Mining	Statistic	0.107	0.086	0.073	0.077	0.123	0.100	0.066
		Ranking	6	4	2	3	7	5	1
3	Food	Statistic	0.107	0.097	0.091	0.101	0.152	0.137	0.105
		Ranking	5	2	1	3	7	6	4
5	Paper	Statistic	0.171	0.130	0.126	0.142	0.146	0.119	0.114
		Ranking	7	4	3	5	6	2	1
6	Chemicals	Statistic	0.183	0.125	0.134	0.153	0.146	0.109	0.120
		Ranking	7	3	4	6	5	1	2
7	Petroleum	Statistic	0.129	0.117	0.127	0.134	0.174	0.116	0.103
		Ranking	5	3	4	6	7	2	1
8	Ceramics	Statistic	0.148	0.104	0.095	0.099	0.117	0.089	0.109
		Ranking	7	4	2	3	6	1	5
9	Basic metal	Statistic	0.181	0.110	0.103	0.142	0.205	0.106	0.116
		Ranking	6	3	1	5	7	2	4
10	Processed metals	Statistic	0.195	0.094	0.079	0.122	0.087	0.082	0.082
		Ranking	7	5	1	6	4	2	3
11	Machinery	Statistic	0.118	0.116	0.120	0.127	0.154	0.131	0.095
		Ranking	3	2	4	5	7	6	1
12	Electrical	Statistic	0.210	0.136	0.112	0.135	0.131	0.114	0.158
		Ranking	7	5	1	4	3	2	6
13	Transport equipment	Statistic	0.128	0.100	0.093	0.100	0.128	0.105	0.085
		Ranking	7	4	2	3	6	5	1
14	Precision instruments	Statistic	0.092	0.085	0.092	0.116	0.158	0.109	0.071
		Ranking	4	2	3	6	7	5	1
15	Other manufacturing	Statistic	0.202	0.126	0.084	0.132	0.076	0.092	0.114
		Ranking	7	5	2	6	1	3	4
16	Construction	Statistic	0.133	0.105	0.090	0.103	0.123	0.095	0.090
		Ranking	7	5	1	4	6	3	2
17	Utilities	Statistic	0.129	0.078	0.065	0.072	0.121	0.067	0.064
		Ranking	7	5	2	4	6	3	1
19	Finance	Statistic	0.208	0.146	0.139	0.150	0.145	0.129	0.134
		Ranking	7	5	3	6	4	1	2
20	Real estate	Statistic	0.130	0.059	0.064	0.062	0.112	0.097	0.082

3. A VES production framework contd..

The VES production function in terms of capital per worker (k) denoted as

$$g(k_t) = Ae^{\mu t}\left[e^{-\left(\frac{k_t-\gamma}{\pi}\right)^{\alpha}} k_t^{\frac{\varepsilon-1}{\varepsilon}} + \left\{1 - e^{-\left(\frac{k_t-\gamma}{\pi}\right)^{\alpha}}\right\} k_t^{\frac{-\beta}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Using $\sigma_{KL} = \frac{dk/k}{dR/R} = \frac{f(k)}{kf\prime(k)}$

I derive $\sigma_{VES} = \frac{g(k_t)}{kg\prime(k_t)} = \frac{\left[\frac{e^{-\left(\frac{k_t-\gamma}{\pi}\right)^{\alpha}}}{1-e^{-\left(\frac{k_t-\gamma}{\pi}\right)^{\alpha}}}\right] k_t^{\frac{\varepsilon+\beta-1}{\varepsilon}} + 1}{\left[\frac{e^{-\left(\frac{k_t-\gamma}{\pi}\right)^{\alpha}}}{1-e^{-\left(\frac{k_t-\gamma}{\pi}\right)^{\alpha}}}\right] k_t^{\frac{\varepsilon+\beta-1}{\varepsilon}} - \frac{\beta}{\varepsilon-1}}$

4. Estimation of structural parameters: CES vs. VES

$$Y = Be^{\gamma t}\left[\delta K^{\frac{\sigma_{CES}-1}{\sigma_{CES}}} + (1-\delta)L^{\frac{\sigma_{CES}-1}{\sigma_{CES}}}\right]^{\frac{\sigma_{CES}}{\sigma_{CES}-1}}$$

CES model parameters ($\boldsymbol{B, \delta, \gamma, \sigma_{CES}}$)

Stage 1. $\ln k_t = \alpha_0 + \alpha_1 \ln\left(\frac{w}{r}\right)_t + \alpha_2 \ln k_{t-1} + u_t$

$$\alpha_0 = \varphi\, \sigma_{CES} \ln \frac{\delta}{1-\delta}$$
$$\alpha_1 = \varphi\, \sigma_{CES}$$
$$\alpha_2 = 1 - \underline{\varphi}.$$

Gives $\hat{\sigma}_{CES}$ and $\hat{\delta}$

Stage 2. $(\Lambda_{CES})_t = \beta_0 + \beta_1 t + v_t$

where $(\Lambda_{CES})_t = \ln\left(\frac{Y}{L}\right)_t - \frac{\hat{\sigma}_{CES}}{\hat{\sigma}_{CES}-1} \ln\left[\hat{\delta} K^{\frac{\hat{\sigma}_{CES}-1}{\hat{\sigma}_{CES}}} + (1-\hat{\delta})\right]$,

$\beta_0 = \log B$, and

$\beta_1 = \gamma$.

Gives $\hat{B}$ and $\hat{\gamma}$

4. Estimation of structural parameters: CES vs. VES contd..

$$V_t = Ae^{\mu t}\left[\theta_t K_t^{\frac{\varepsilon-1}{\varepsilon}} + (1-\theta_t)\left(\frac{K}{L}\right)_t^{\frac{-\beta}{\varepsilon}} L_t^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

VES model parameters ($\mathbf{A}, \boldsymbol{\mu}, \boldsymbol{\beta}, \boldsymbol{\varepsilon}$)

Stage 1. $log\frac{Y}{L} = \alpha + \varepsilon logW + \beta log\frac{K}{L} + u$

Gives $\hat{\varepsilon}$ and $\hat{\beta}$

Stage 2. Either (a) estimate time-variant $\theta_t \simeq F(Weibull) = e^{-\left(\frac{\mathrm{k}_t-\gamma}{\pi}\right)^{\alpha}}$

Or, (b) estimate time-invariant θ using

$z1_t = \Psi_0 + \Psi_1\, z2_t + u_t$, where $z1_t = \left(\frac{Y}{K}\right)^{\frac{\hat{\varepsilon}-1}{\hat{\varepsilon}}}$ and $z2_t = k^{\frac{\hat{\varepsilon}+\widehat{\beta}-1}{\hat{\varepsilon}}}$.

$\theta = \frac{1}{1+\frac{\Psi_0}{\Psi_1}}$ (Lu and Fletcher, 1968)

Gives $\widehat{\theta_t}$ and $\hat{\theta}$

4. Estimation of structural parameters: CES vs. VES contd..

Stage 2. contd.. Two estimates of the share-parameter

$$\theta_t = e^{-\left(\frac{\mathrm{k}_t-\gamma}{\pi}\right)^{\alpha}} \text{ VES-W model and } \theta = \frac{1}{1+\frac{\Psi_0}{\Psi_1}} \text{ VES model,}$$

Consequently,

$$\hat{\boldsymbol{\sigma}}_{\boldsymbol{VES}}\Big|_{\boldsymbol{\theta_t = e^{-\left(\frac{k_t-\gamma}{\pi}\right)^{\alpha}}}} = \frac{\left[\frac{e^{-\left(\frac{\mathrm{k}_t-\gamma}{\pi}\right)^{\alpha}}}{1-e^{-\left(\frac{\mathrm{k}_t-\gamma}{\pi}\right)^{\alpha}}}\right] k_t^{\frac{\hat{\varepsilon}+\hat{\beta}-1}{\hat{\varepsilon}}} + 1}{\left[\frac{e^{-\left(\frac{\mathrm{k}_t-\gamma}{\pi}\right)^{\alpha}}}{1-e^{-\left(\frac{\mathrm{k}_t-\gamma}{\pi}\right)^{\alpha}}}\right] k_t^{\frac{\hat{\varepsilon}+\hat{\beta}-1}{\hat{\varepsilon}}} - \frac{\hat{\beta}}{\hat{\varepsilon}-1}} \quad \textbf{or} \quad \hat{\boldsymbol{\sigma}}_{\boldsymbol{VES}}\Big|_{\boldsymbol{\theta=\hat{\theta}}} = \frac{\left[\frac{\hat{\theta}}{1-\hat{\theta}}\right] k_t^{\frac{\hat{\varepsilon}+\hat{\beta}-1}{\hat{\varepsilon}}} + 1}{\left[\frac{\hat{\theta}}{1-\hat{\theta}}\right] k_t^{\frac{\hat{\varepsilon}+\hat{\beta}-1}{\hat{\varepsilon}}} - \frac{\hat{\beta}}{\hat{\varepsilon}-1}}$$

Stage 3. Technological parameters, A and μ, are estimated using the estimates of θ, ε and β and σ_{VES} from stages 1 and 2.

Gives $\hat{A}$ and $\hat{\mu}$

4. Estimation of structural parameters: CES vs. VES contd..

CES model results (stage 1)

$$ln\, k_t = \alpha_0 + \alpha_1\, ln\left(\frac{w}{r}\right)_t + \alpha_2\, ln\, k_{t-1} + u_t.$$

Industry code	Industry name	$\hat{\alpha}_1$ (coefficient of log$\frac{w}{r}$)	$\hat{\alpha}_2$ (coefficient of log$\frac{K}{L}$)	Constant term	R^2	Durbin-Watson statistic	$\hat{\sigma}_{CES}$
1	Agriculture	0.545***	0.613***	-0.325**	0.913	16.95	1.41
2	Mining	0.513***	0.348***	0.279	0.609	13.78	0.79
3	Food	0.523***	0.317***	0.916***	0.863	25.19	0.77
5	Paper	0.694***	0.288***	-0.178	0.928	8.92	0.97
6	Chemicals	0.856***	0.288**	-0.210	0.863	18.69	1.20
7	Petroleum	0.943***	0.318***	0.936***	0.931	13.14	1.38
8	Ceramics	0.821***	0.306***	-0.763***	0.878	14.93	1.18
9	Basic metal	0.834***	0.304***	-0.604*	0.798	8.514	1.20
10	Processed metals	-0.294*	0.587***	1.145**	0.498	12.05	-0.71
11	Machinery	0.643***	0.424***	-0.800**	0.717	9.83	1.12
12	Electrical	0.477***	0.549***	-0.350	0.825	21.84	1.06
13	Transport equipment	0.720***	0.339***	-0.556**	0.804	12.09	1.09
14	Precision instruments	0.640***	0.578***	-1.040***	0.824	14.59	1.52
15	Other manufacturing	0.441***	0.467***	-0.477**	0.772	10.49	0.83
16	Construction	0.194*	0.635***	-0.140	0.622	22.12	0.53
17	Utilities	0.846***	0.184**	0.601**	0.875	26.51	1.04
19	Finance	0.363***	0.668***	-0.120	0.876	29.92	1.09
20	Real estate	0.098	0.520***	1.349**	0.241	9.30	0.20
21	Transport	1.118***	0.510***	-2.412***	0.949	13.99	2.28
22	Non-govt services	0.298**	0.823***	-0.519*	0.908	24.01	1.68

Source: Author's calculation based on the Regional-level Japanese Industrial Productivity (R-JIP) database

4. Estimation of structural parameters: CES vs. VES contd..

VES model results (stage 1)

$$\log\frac{Y}{L} = \alpha + \varepsilon \log W + \beta \log\frac{K}{L} + u$$

Industry code	Industry name	$\hat{\varepsilon}$	$\hat{\beta}$	R2	$\theta = \hat{\theta}$	$\bar{\theta}_t = Avg\left[e^{-\left(\frac{k_t-\gamma}{\pi}\right)^{\alpha}}\right]$
1	Agriculture	0.809***	0.164***	0.950	0.676	0.526
2	Mining	0.900***	0.262***	0.973	0.749	0.623
3	Food	0.721***	0.149***	0.970	0.926	0.740
5	Paper	0.862***	0.143***	0.989	0.710	0.608
6	Chemicals	0.871***	0.322***	0.984	0.459	0.697
7	Petroleum	0.528***	0.394***	0.974	0.852	0.539
8	Ceramics	0.842***	0.173***	0.995	0.585	0.507
9	Basic metal	0.922***	0.263***	0.988	0.270	0.552
10	Processed metals	0.891***	0.119***	0.994	0.450	0.850
11	Machinery	0.956***	0.185***	0.991	0.328	0.506
12	Electrical	0.920***	0.254***	0.988	0.434	0.590
13	Transport equipment	0.927***	0.234***	0.987	0.327	0.694
14	Precision instruments	0.853***	0.137***	0.995	0.487	0.547
15	Other manufacturing	0.923***	0.172***	0.997	0.557	0.735
16	Construction	0.927***	0.152***	0.994	0.581	0.577
17	Utilities	1.190***	0.119**	0.952	0.835	0.661
19	Finance	0.780***	0.367***	0.992	0.163	0.600
20	Real estate	0.829***	0.082	0.835	0.997	0.805
21	Transport	0.835***	0.077***	0.996	0.104	0.429
22	Non-govt services	0.949***	0.159***	0.998	0.482	0.634

Average of $\hat{\theta}$ → **.55** **.62**

Source: Author's calculation based on the Regional-level Japanese Industrial Productivity (R-JIP) database

4. Estimation of structural parameters: CES vs. VES contd..

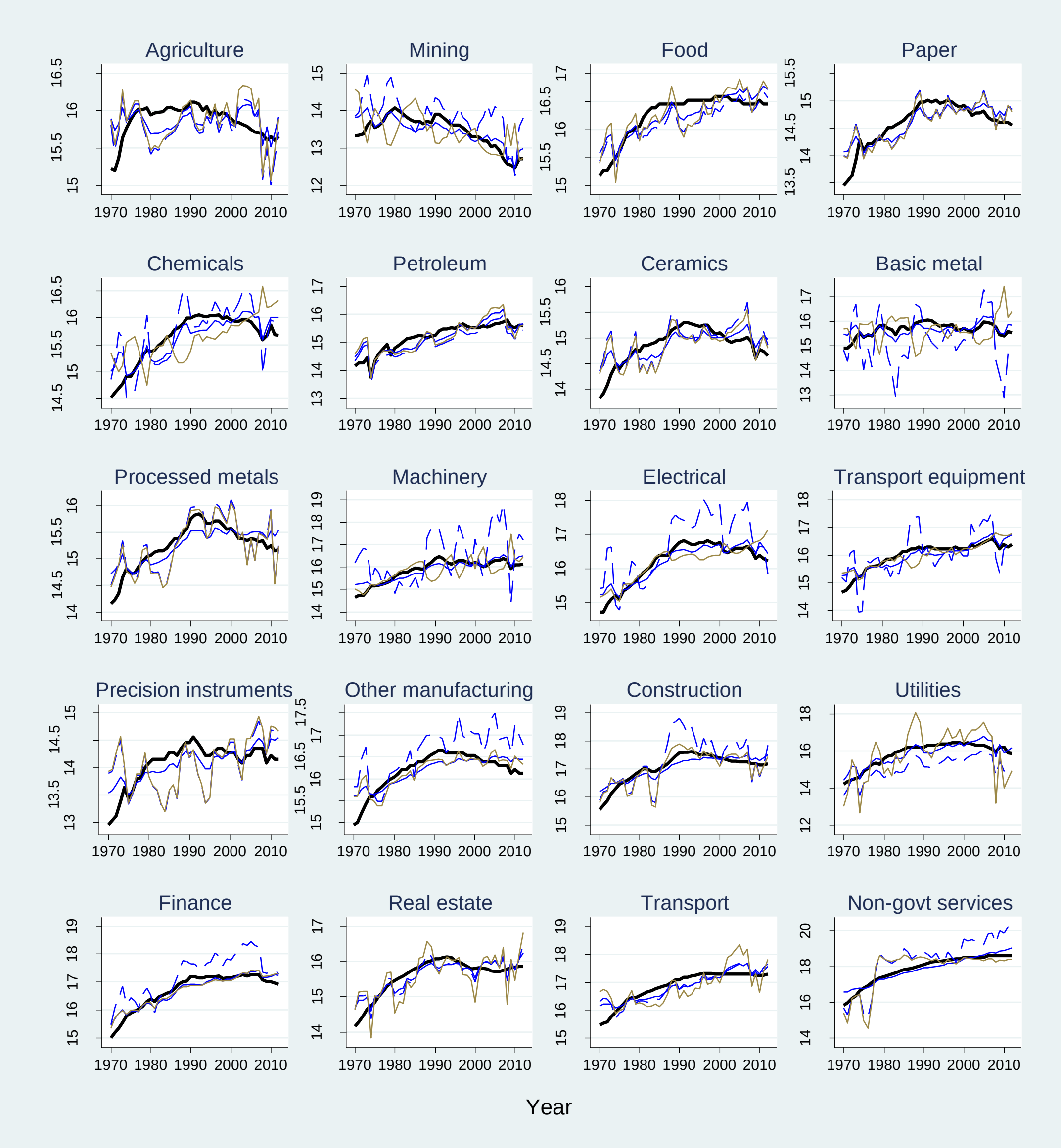
Industry code	Industry name	CES $\hat{\sigma}_{CES}$	VES $\hat{\sigma}_{VES}\|_{\theta=\hat{\theta}}$	VES-W $\hat{\sigma}_{VES}\|_{\theta_t=e^{-\left(\frac{k_t-\gamma}{\pi}\right)^{\alpha}}}$
1	Agriculture	1.408	1.050	1.075
2	Mining	0.787	0.782	0.714
3	Food	0.766	1.061	1.215
5	Paper	0.975	0.990	0.986
6	Chemicals	1.202	0.655	0.802
7	Petroleum	1.383	1.052	1.116
8	Ceramics	1.183	0.963	0.956
9	Basic metal	1.198	0.416	0.610
10	Processed metals	-0.712	0.952	0.987
11	Machinery	1.116	0.337	0.485
12	Electrical	1.058	0.496	0.633
13	Transport equipment	1.089	0.441	0.697
14	Precision instruments	1.517	1.036	1.032
15	Other manufacturing	0.827	0.663	0.792
16	Construction	0.532	0.699	0.723
17	Utilities	1.037	1.101	12.838
19	Finance	1.093	0.666	0.840
20	Real estate	0.204	1.000	1.162
21	Transport	2.282	1.953	1.556
22	Non-govt services	1.684	0.488	0.627
Average of $\hat{\sigma}$ →		**1.03**	**.84**	**.91**

Note:

The figures for the VES models are averages over the period from 1970-2012.

Source: Author's calculation based on the Regional-level Japanese Industrial Productivity (R-JIP) database

5. Descriptive accuracy of the models (log output level)



Note:

The **solid black** lines (actual value-added), the **solid-blue lines** (CES model), **dashed-blue lines** (VES) and **brown lines** (VES-W) show calibrated value-added figures.

The Kormogorov-Smirnov test for goodness of fit suggests that both the CES and the VES-W fit actual output level reasonably well.

Source: Author's calculation based on the Regional-level Japanese Industrial Productivity (R-JIP) database

5. Descriptive accuracy of the models (labor income share)

Industry code	Industry name	Average labor income share, 1970-2012			
		Actual	CES	VES	VES-Weibull
1	Agriculture	0.536	0.551	0.61	0.57
2	Mining	0.548	0.566	0.398*	0.598
3	Food	0.314	0.325	0.376*	0.333
5	Paper	0.581	0.599*	0.591*	0.604*
6	Chemicals	0.416	0.43	0.415*	0.446
7	Petroleum	0.067	0.071	0.085	0.074
8	Ceramics	0.641	0.667*	0.653	0.689*
9	Basic metal	0.552	0.573*	1.492*	0.641*
10	Processed metals	0.821	0.843	0.853*	0.879
11	Machinery	0.738	0.76	0.674*	0.8
12	Electrical	0.65	0.666	0.485*	0.673
13	Transport equipment	0.639	0.657*	0.757*	0.66
14	Precision instruments	0.749	0.782*	0.917*	0.897*
15	Other manufacturing	0.79	0.815*	0.555*	0.823*
16	Construction	0.832	0.851*	0.771*	0.926
17	Utilities	0.297	0.304	0.666*	0.596*
19	Finance	0.533	0.55*	0.353*	0.548
20	Real estate	0.414	0.422	0.43	0.469
21	Transport	0.712	0.751	0.762*	0.836*
22	Non-govt services	0.822	0.845*	0.762*	1.188
How many * →			**9/20**	**16/20**	**7/20**

Note:

The Kormogorov-Smirnov test for goodness of fit is performed for each calibrated distribution with the actual one.

* indicates the case when two distributions are different at 1% level of statistical significance.

Source: Author's calculation based on the Regional-level Japanese Industrial Productivity (R-JIP) database

Concluding remarks

This paper develops a VES framework as an alternative to the CES model to analyse movements in the labor income share.

The VES model embodies a variable elasticity of substitution (σ_{KL}) and a share parameter (θ_t), both as non-linear functions of the capital-labor ratio.

Empirical findings support the choice of a variable elasticity of substitution.

The movements in factor income are reasonably reflected in both the CES and VES models, **BUT the role of Accumulation principle in labor income share dynamics is more consistent with the VES model**.